Analysis of Combined Free and Forced Convection Film Boiling
Part I: Forced and Free Convection Regions

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INTRODUCTION

The so-called two phase boundary layer theory was successfully employed by Cess and Sparrow (1961) for forced convection film boiling on a horizontal flat plate, and was subsequently adopted by Koh (1962) for free convection film boiling on a vertical flat plate. Using an integral method similar to the one employed for film condensation (Jacobs, 1966), Jacobs and Boehm (1970) attacked film boiling in the presence of both a body force and forced convection, in which similarity solutions no longer exist. Unfortunately, it was pointed out by Fujii et al. (1971) that the boundary condition for the liquid phase used by Jacobs and Boehm was incorrect, and hence, the asymptote for pure free convection film boiling differs from the similarity solution.

It is the purpose here in Part I to provide a consistent integral treatment for combined free and forced convection film boiling, which naturally leads to the correct asymptotes for pure forced convection film boiling and for pure free convection film boiling. A detailed solution procedure for the mixed convection region, where the effects of both a body force and forced convection are equally important, is discussed in Part II (Nakayama and Koyama).

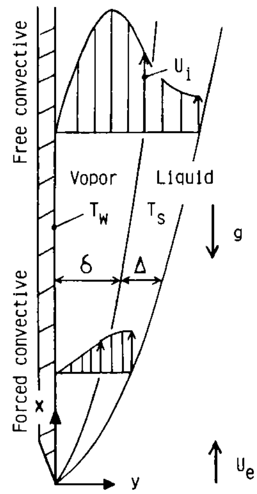

Figure 1. Physical model and coordinates.

INTEGRAL TREATMENT

The physical model considered for study is shown schematically in Figure 1. A vertical flat plate heated to a constant temperature T_w is placed in a saturated liquid of temperature T_s flowing at a constant velocity u_e. In common with previous studies of film boiling and condensation, it has been necessary to omit interfacial waviness and film instability in order to make the problem tractable within the scope of the integral approach based on the boundary layer approximations. Thus, the vapor layer of thickness δ and the liquid layer of thickness Δ are assumed to be stable and smooth. As illustrated in Figure 1, the buoyancy force is small near the lead edge, and the external forced flow essentially drives the vapor flow there; this is the forced convection region. Far downstream, on the other hand, the buoyancy force becomes so significant that the vapor velocity level may even exceed the external liquid flow velocity level, and thus the predominant driving force there is the buoyancy force; this is the free convection region. It should, however, be noted that during actual film boiling, wave formation film instability, and transition to turbulence may be observed. Therefore, a vapor flowing over a flat plate in forced convection may never actually experience the subsequent transition from the forced convection region to the free convection region. The pure forced convection and pure free convection solutions should rather be regarded as the two distinct asymptotic results of combined free and forced convection film boiling treated in Part II, namely, the solution without a gravity force and the solution without forced convection (i.e., pool film boiling), respectively.

Koh (1962) studied free convection film boiling, and showed that neglect of the inertia and convection terms in the vapor phase results in very little loss of accuracy for the vapor Prandtl number $Pr > 1$. A similar conclusion was achieved by Cess and Sparrow (1961) for forced convection film boiling. Thus, the momentum and energy equations for the vapor phase may be approximated as

$$\nu \frac{\partial^2 u}{\partial y^2} + g' = 0 \tag{1a}$$

$$\frac{\partial^2 T}{\partial y^2} = 0 \quad (1b)$$

where

$$g' \equiv g(\rho_f - \rho)/\rho \quad (1c)$$

The subscript f refers to the liquid phase while no subscript is assigned for the vapor phase. Integration of Eqs. 1a and 1b yields

$$u/u_i = (1 + \Lambda)\eta - \Lambda\eta^2 \quad (2a)$$

and

$$(T - T_s)/(T_w - T_s) = 1 - \eta \quad (2b)$$

where

$$\Lambda \equiv g'\delta^2/2\nu u_i \quad (2c)$$

and

$$\eta = y/\delta \quad (2d)$$

Λ is a velocity shape factor and u_i, the interfacial velocity. For the liquid boundary layer, on the other hand, the velocity profile is assumed to follow

$$u/u_e \equiv f(\eta_f; u_i^*) = u_i^* + (1 - u_i^*)(2\eta_f - \eta_f^2) \quad (3a)$$

where

$$\eta_f \equiv (y - \delta)/\Delta \quad (3b)$$

and

$$u_i^* \equiv u_i/u_e \quad (3c)$$

A usual control volume consideration within the liquid layer leads to

$$\frac{d}{dx}\int_\delta^{\delta+\Delta} \rho_f(u_e u - u^2)\,dy + (u_e - u_i)\frac{d}{dx}\int_o^\delta \rho u\,dy = \mu_f \left.\frac{\partial u}{\partial y}\right|_{y=\delta} \quad (4a)$$

The second term on the lefthand side is usually negligibly small since the density ratio ρ/ρ_f is naturally very small. Thus,

$$\frac{d}{dx}\int_\delta^{\delta+\Delta} \rho_f(u_e u - u^2)\,dy \cong \mu_f \left.\frac{\partial u}{\partial y}\right|_{y=\delta} \quad (4b)$$

An analogous simplifying assumption was adopted by Shekliladze and Gomelauri (1966) for the analysis of forced convection film condensation. Now, the energy balance relation along the liquid-vapor interface may be given by

$$h_{fg}\frac{d}{dx}\int_o^\delta \rho\, u\,dy = -k\left.\frac{\partial T}{\partial y}\right|_{y=\delta} \quad (5)$$

The proposed profiles given by Eqs. 2a, 2b, and 3a are substituted into the integral equations 4b and 5. Upon carrying out the integrations as well as differentiations with respect to η and η_f, Eqs. 4b and 5 may be transformed into a pair of ordinary differential equations for Δ^2 and δ^2, which yields

$$(\Delta/x)^2\, Rex = \frac{30}{\left(1 + \frac{3}{2}u_i^*\right)} I_f \quad (6a)$$

and

$$(\delta/x)^2\, Rex = \frac{4}{u_i^*\left(1 + \frac{\Lambda}{3}\right)} I \left(\frac{H}{Pr}\right)\left(\frac{\nu}{\nu_f}\right) \quad (6b)$$

where

$$Rex \equiv u_e x/\nu_f, \qquad H \equiv C_p(T_w - T_s)/h_{fg} \quad (6c)$$

$$I_f = \frac{\int_o^x (1 - u_i^*)^2\left(1 + \frac{3}{2}u_i^*\right)dx}{(1 - u_i^*)^2\left(1 + \frac{3}{2}u_i^*\right)x} \quad (6d)$$

and

$$I = \frac{\int_o^x (3 + \Lambda)u_i^* dx}{(3 + \Lambda)\,u_i^* x} \quad (6e)$$

It is noted that the Reynolds number Rex, as used by Cess and Sparrow (1961), is based on the kinematic viscosity of the liquid phase. The matching condition for the interfacial shear is given by

$$\mu\left.\frac{\partial u}{\partial y}\right|_{y=\delta} = \mu_f\left.\frac{\partial u}{\partial y}\right|_{y=\delta} \quad (7a)$$

namely,

$$\Delta/\delta = 2\,(\mu_f/\mu)\frac{1 - u_i^*}{u_i^*(1 - \Lambda)} \quad (7b)$$

The substitution of Eqs. 6a and 6b into Eq. 7b leads to

$$\frac{H}{PrR} = \frac{15}{8}\,\frac{u_i^{*3}(1 - \Lambda)^2\left(1 + \frac{\Lambda}{3}\right)}{(1 - u_i^*)^2\left(1 + \frac{3}{2}u_i^*\right)}\,\frac{I_f}{I} \quad (8a)$$

where

$$R \equiv \rho\mu/(\rho\mu)_f \quad (8b)$$

Furthermore, Eqs. 2c and 6b are combined to give

$$u_i^{*2} = \frac{2x^*}{\Lambda\left(1 + \frac{\Lambda}{3}\right)} I \quad (9a)$$

where

$$x^* = \left(\frac{H}{Pr}\right)\left(\frac{g'x}{u_e^2}\right) \quad (9b)$$

Thus, Eqs. 9a and 8a form a pair of characteristic equations which can be solved for two unknowns $u_i^*(x^*)$ and $\Lambda(x^*)$ as a single lumped parameter H/PrR is specified. Once u_i^* and Λ are determined, the local Nusselt number $Nux = x/\delta$ of the present concern may be evaluated from the following expression reduced from Eq. 6b:

$$Nux/(RexPrR/4H)^{1/2}(\mu_f/\mu) = \left[u_i^*\left(1 + \frac{\Lambda}{3}\right)\Big/ I\right]^{1/2} \quad (10a)$$

An equivalent expression may be obtained utilizing Eq. 9:

$$Nux/(GrxPr/16H)^{1/4} = \left[2\left(1 + \frac{\Lambda}{3}\right)\Big/ \Lambda I\right]^{1/4} \quad (10b)$$

where

$$Grx \equiv g'x^3/\nu^2 \quad (10c)$$

Asymptotic Results for Forced Convection Region

The perturbation around $x^* = 0$ suggests

$$u_i^* = u_i^*(0) + o(x^{*2}) \quad (11a)$$

and

$$\Lambda = [2/(u_i^*(0))^2]\,x^* + o(x^{*2}) \quad (11b)$$

Consequently, the functions I and I_t defined by Eqs. 6e and 6d become unity in the forced convection region. The ratio of the interfacial velocity to the free stream velocity u_i^* may readily be determined from the following simple algebraic equation, which is reduced from Eq. 8a:

$$\frac{H}{PrR} = \frac{15}{8}\,\frac{u_i^{*3}}{(1 - u_i^*)^2\left(1 + \frac{3}{2}u_i^*\right)} \tag{12}$$

The above equation indicates that u_i^* approaches unity as H/PrR goes to infinity. As clearly seen in Figure 2a, the local Nusselt number variation based on Eqs. 12 and 10a agrees extremely well with the Cess and Sparrow (1961) exact solution. The local friction coefficient evaluated from

$$(2\tau_w/\rho_f u_e^2)\,(RexH/PrR)^{1/2} = u_i^{*3/2} \tag{13}$$

is also plotted in Figure 2b, which again shows an excellent agreement between the exact solution and the present approximate solution. As may be expected, the formation of the vapor film leads to a significant reduction in a drag force.

Asymptotic Results for Free Convection Region

The consideration of the asymptotic condition, namely, $x^* >> 1$, reveals the constancy of Λ, and the proportional relationship, namely, $x^{*2} \propto u_i^* >> 1$. Equations 6d and 6e then yield $I = 2/3$ and $I_f = 2/5$, and subsequently Eqs. 9a and 8a reduce to

$$u_i^* = \left[\frac{4}{\Lambda(3 + \Lambda)}\,x^*\right]^{1/2} \tag{14a}$$

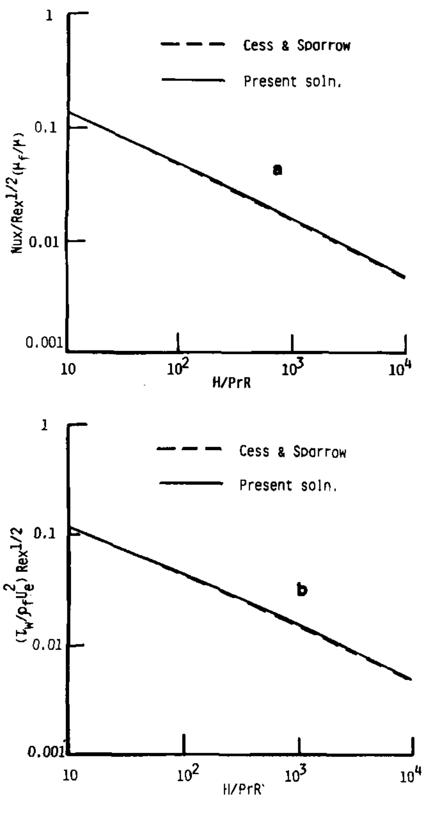

Figure 2. Results for pure forced convection film boiling. (a) Local Nusselt number. (b) Local friction coefficient.

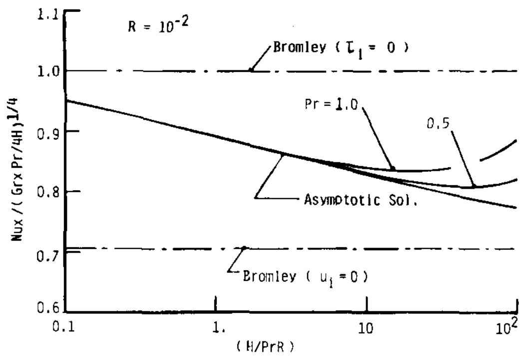

Figure 3. Heat transfer results for pure free convection film boiling.

and

$$\frac{H}{PrR} = \frac{1}{4}(3 + \Lambda)(1 - \Lambda)^2 \tag{14b}$$

The shape factor Λ may readily be determined from Eq. 14b for a given lumped parameter H/PrR. Since Λ increases from unity as increasing H/PrR, the righthand side of Eq. 10b for the Nux expression approaches unity for $H/PrR >> 1$. The asymptotic curve generated using Eqs. 10b and 14b is plotted in Figure 3 along with the Koh exact solutions. Since the effect of convection becomes significant for a thick vapor layer, the exact solutions for $R = 10^{-2}$ are seen to depart from the present solution as H/PrR increases such that $H/Pr >> 1$. However, the exact solutions for the smaller R, namely, $R = 10^{-6}$, turn out to be indistinguishable from the present asymptotic curve within this range of H/PrR.

CONCLUSION

Usually, approximate methods like the present one are designed primarily for a quick estimate of the boundary layer characteristic quantities (such as friction coefficient, heat transfer coefficient, and related factors), and one does not really expect an approximate method to give accurate velocity and temperature distributions within the boundary layers. Nevertheless, it would be worthwhile to check the predicted profiles against the exact solutions so that the accuracy of the procedure may be examined.

The velocity and temperature profiles are plotted in Figure 4 with the exact solutions (Koh, 1962) for the case of relatively thin vapor film. The abscissa variables in the figures are chosen as the similarity variables used by Koh, namely, $\eta_V = (y/x)(Grx/4)^{1/4}$ and $\eta_F = \frac{y - \delta}{x}(\nu/\nu_f)^{1/2}(Grx/4)^{1/4}$. Even the details of the profiles are seen to agree closely with the exact solutions. This fact substatiates the validity of the present integral formulation.

NOTATION

C_p	= specific heat of vapor
g, g'	= acceleration due to gravity, g' defined by Eq. 1c
Grx	= Grashof number
H	= sensible-latent heat ratio
h_{fg}	= latent heat of vaporization
I, I_f	= functions defined by Eqs. 6e and 6d

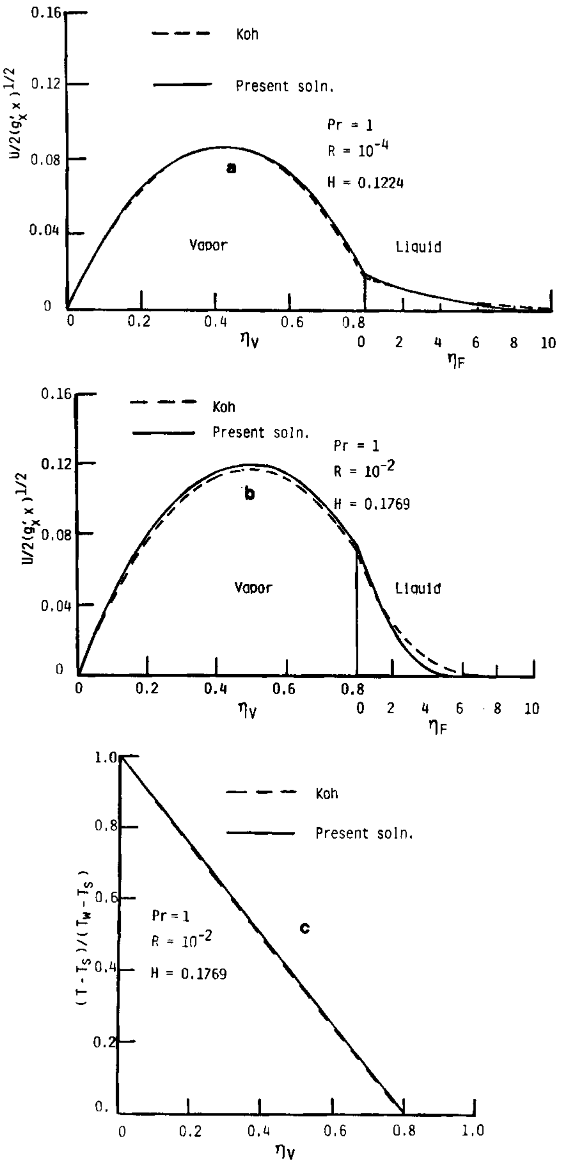

Figure 4. Velocity and temperature profiles in pure free convection film boiling. (a) Velocity profile, $R = 10^{-4}$. (b) Velocity profile, $R = 10^{-2}$. (c) Temperature profile, $R = 10^{-2}$.

k = thermal conductivity of vapor
Nux = local Nusselt number
Pr = Prandtl number of vapor
R = density-viscosity ratio
Rex = Reynolds number
T = temperature
u = streamwise velocity component
u_i, u_i^* = interfacial velocity, $u_i^* = u_i/u_e$
x, (x^*), y = boundary layer coordinates, x^* defined by Eq. 9b

Greek Letters

δ = vapor layer thickness
Δ = liquid layer thickness
η, η_t = similarity variables
Λ = velocity shape factor defined by Eq. 2c
μ, ν = viscosity, kinematic viscosity
ρ = density

Subscripts

e = free stream
f = liquid
w = wall

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